

BINOMIAL DISTRIBUTION

1) $n = 10$

$r = 0, 1$ (knock down)

$p = \frac{1}{6}$ (knock down)

$q = \frac{5}{6}$

 $P(\text{knock down less than 2 hurdles})$

$= P(0) + P(1)$

$= {}^{10}C_0 \left(\frac{1}{6}\right)^0 \left(\frac{5}{6}\right)^{10} + {}^{10}C_1 \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^9$

$= 0.485$

2) (i) $n = 8$

$r = 2$ (getting 3)

$p = \frac{1}{6}$ (---)

$q = \frac{5}{6}$

 $P(\text{getting 3 exactly twice})$

$= P(2)$

$= {}^8C_2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^6$

$= 0.260$

(ii) $n = 8$

$r = 7, 8$ (getting 3)

$p = \frac{1}{6}$ (---)

$q = \frac{5}{6}$

 $P(\text{getting 3 atleast 7 times})$

$= P(7) + P(8)$

$= {}^8C_7 \left(\frac{1}{6}\right)^7 \left(\frac{5}{6}\right)^1 + {}^8C_8 \left(\frac{1}{6}\right)^8 \left(\frac{5}{6}\right)^0$

$= 2.441 \times 10^{-5}$

(iii) $n = 8$

$r = 1, 2, \dots, 8$ (getting 3)

$p = \frac{1}{6}$ (---)

$q = \frac{5}{6}$

 $P(\text{getting 3 atleast once})$

$= P(1) + P(2) + \dots + P(8)$

$= 1 - P(0)$

$= 1 - {}^8C_0 \left(\frac{1}{6}\right)^0 \left(\frac{5}{6}\right)^8$

$= 0.767$

3) $n = 7$

$r = 0, 1$ (defective)

$p = 0.1$ (---)

$q = 0.9$

 $P(\text{Not more than 1 defective})$

$= P(0) + P(1)$

$= {}^7C_0 (0.1)^0 (0.9)^7 +$

${}^7C_1 (0.1)^1 (0.9)^6$

$= 0.850$

4) (i) $N = 100$

$n = 10$

$r = 7$ (heads)

$p = \frac{1}{2}$ (---)

$q = \frac{1}{2}$

 $P(7 \text{ heads \& 3 tails})$

$= {}^{10}C_7 \left(\frac{1}{2}\right)^7 \left(\frac{1}{2}\right)^3$

$= 0.1172$

No. of sets with 7 heads & 3 tails

$= N \times P$

$= 100 \times 0.1172$

$= 117.2$

≈ 117

$$\begin{aligned} \text{ii) } N &= 100 \\ n &= 10 \\ r &= 7, 8, 9, 10 \text{ (heads)} \\ p &= \frac{1}{2} \text{ (---"---)} \\ q &= \frac{1}{2} \end{aligned}$$

$$\begin{aligned} P(\text{at least 7 heads}) &= P(7) + P(8) + P(9) + P(10) \\ &= {}^{10}C_7 \left(\frac{1}{2}\right)^7 \left(\frac{1}{2}\right)^3 + {}^{10}C_8 \left(\frac{1}{2}\right)^8 \left(\frac{1}{2}\right)^2 \\ &\quad + {}^{10}C_9 \left(\frac{1}{2}\right)^9 \left(\frac{1}{2}\right)^1 + {}^{10}C_{10} \left(\frac{1}{2}\right)^{10} \left(\frac{1}{2}\right)^0 \\ &= 0.1719 \end{aligned}$$

$$\begin{aligned} \text{No. of sets with at least 7 heads} &= N \times P \\ &= 100 \times 0.1719 \\ &= 171.9 \\ &\approx 172 \end{aligned}$$

$$\begin{aligned} \text{5)(i) } N &= 2000 \\ n &= 4 \\ r &= 1, 2, 3, 4 \text{ (boy)} \\ p &= \frac{1}{2} \text{ (boy)} \\ q &= \frac{1}{2} \end{aligned}$$

$$\begin{aligned} P(\text{at least a boy}) &= P(1) + P(2) + P(3) + P(4) \\ &= {}^4C_1 \left(\frac{1}{2}\right) \left(\frac{1}{2}\right)^3 + {}^4C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^2 \\ &\quad + {}^4C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^1 + {}^4C_4 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^0 \\ &= 0.9375 \end{aligned}$$

$$\begin{aligned} \text{No. of families with at least a boy} &= N \times P \\ &= 2000 \times 0.9375 = 1875 \end{aligned}$$

$$\begin{aligned} \text{ii) } N &= 2000 \\ n &= 4 \\ r &= 1, 2 \text{ (girl)} \\ p &= \frac{1}{2} \text{ (girl)} \\ q &= \frac{1}{2} \end{aligned} \quad \textcircled{2}$$

$$\begin{aligned} P(1 \text{ or } 2 \text{ girl}) &= P(1) + P(2) \\ &= {}^4C_1 \left(\frac{1}{2}\right) \left(\frac{1}{2}\right)^3 + {}^4C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^2 \\ &= 0.625 \end{aligned}$$

$$\begin{aligned} \text{No. of families with 1 or 2 girls} &= N \times P \\ &= 2000 \times 0.625 \\ &= 1250 \end{aligned}$$

$$\begin{aligned} \text{iii) } N &= 2000 \\ n &= 4 \\ r &= 0 \text{ (girl)} \\ p &= \frac{1}{2} \text{ (girl)} \\ q &= \frac{1}{2} \end{aligned}$$

$$\begin{aligned} P(\text{No girl}) &= P(0) \\ &= {}^4C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^4 \\ &= 0.0625 \end{aligned}$$

$$\begin{aligned} \text{No. of families with 0 girl} &= N \times P \\ &= 2000 \times 0.0625 \\ &= 125 \end{aligned}$$

$$\begin{aligned} \text{(iv) } N &= 2000 \\ n &= 4 \\ r &= 2 \text{ (boys)} \\ p &= \frac{1}{2} \text{ (---)} \quad q = \frac{1}{2} \\ P(2 \text{ boys}) &= P(2) = {}^4C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^2 \\ &= 0.375 \end{aligned}$$

$$\begin{aligned} \text{No. of families with 2 boys} &= N \times P = 2000 \times 0.375 \\ &= 750 \end{aligned}$$

$$6) (i) n = 6$$

$$r = 3, 4, 5, 6 \text{ (busy)}$$

$$p = \frac{1}{15} \text{ (busy)}$$

$$q = \frac{14}{15}$$

$$P(\text{at least 3 busy})$$

$$= P(3) + P(4) + P(5) + P(6)$$

$$= {}^6C_3 \left(\frac{1}{15}\right)^3 \left(\frac{14}{15}\right)^3 + {}^6C_4 \left(\frac{1}{15}\right)^4 \left(\frac{14}{15}\right)^2$$

$$+ {}^6C_5 \left(\frac{1}{15}\right)^5 \left(\frac{14}{15}\right)^1 + {}^6C_6 \left(\frac{1}{15}\right)^6 \left(\frac{14}{15}\right)^0$$

$$= 5.084 \times 10^{-3}$$

$$ii) n = 6$$

$$r = 0, 1, 2, 3 \text{ (busy)}$$

$$p = \frac{1}{15} \text{ (busy)}$$

$$q = \frac{14}{15}$$

$$P(\text{Not more than 3 busy})$$

$$= P(0) + P(1) + P(2) + P(3)$$

$$= {}^6C_0 \left(\frac{1}{15}\right)^0 \left(\frac{14}{15}\right)^6 + {}^6C_1 \left(\frac{1}{15}\right)^1 \left(\frac{14}{15}\right)^5$$

$$+ {}^6C_2 \left(\frac{1}{15}\right)^2 \left(\frac{14}{15}\right)^4 + {}^6C_3 \left(\frac{1}{15}\right)^3 \left(\frac{14}{15}\right)^3$$

$$= 0.9997$$

$$7) n = 10$$

$$r = 7, 8, 9, 10 \text{ (head)}$$

$$p = \frac{1}{2} \text{ (head)}$$

$$q = \frac{1}{2}$$

$$P(\text{at least 7 heads}) = P(7) + \dots + P(10)$$

$$= {}^{10}C_7 \left(\frac{1}{2}\right)^{10} + {}^{10}C_8 \left(\frac{1}{2}\right)^{10} + {}^{10}C_9 \left(\frac{1}{2}\right)^{10}$$

$$+ {}^{10}C_{10} \left(\frac{1}{2}\right)^{10}$$

$$= 0.1719$$

$$8) P(A \text{ win}) = \frac{3}{5}$$

$$P(B \text{ win}) = \frac{2}{5}$$

$$n = 5$$

$$r = 3, 4, 5 \text{ (A's win)}$$

$$p = \frac{3}{5} \text{ (---)}$$

$$q = 1 - p = \frac{2}{5}$$

$$P(A \text{ win at least 3 out of 5})$$

$$= P(3) + P(4) + P(5)$$

$$= {}^5C_3 \left(\frac{3}{5}\right)^3 \left(\frac{2}{5}\right)^2 + {}^5C_4 \left(\frac{3}{5}\right)^4 \left(\frac{2}{5}\right)^1$$

$$+ {}^5C_5 \left(\frac{3}{5}\right)^5 \left(\frac{2}{5}\right)^0$$

$$= 0.6826$$

$$9) (i) n = 6$$

$$r = 5, 6$$

$$p = 0.75$$

$$q = 0.25$$

$$P(\text{claim accepted}) = P(5) + P(6)$$

$$= {}^6C_5 (0.75)^5 (0.25)^1$$

$$+ {}^6C_6 (0.75)^6 (0.25)^0$$

$$= 0.5339$$

$$(ii) P(\text{claim rejected})$$

$$= 1 - P(\text{claim accepted})$$

$$= 1 - 0.5339$$

$$= 0.4661$$

$$10) n = 8$$

$$r = 6, 7, 8 \text{ (correct)}$$

$$p = \frac{1}{3} \quad q = \frac{2}{3}$$

$$P(\text{at least 6 correct}) = P(6) + P(7) + P(8)$$

$$= {}^8C_6 \left(\frac{1}{3}\right)^6 \left(\frac{2}{3}\right)^2 + {}^8C_7 \left(\frac{1}{3}\right)^7 \left(\frac{2}{3}\right)^1 + {}^8C_8 \left(\frac{1}{3}\right)^8$$

$$= 0.0197$$

$$\begin{aligned}
 (11) \quad P(5 \text{ even}) &= 2 P(4 \text{ even}) \\
 {}^{10}C_5 p^5 q^5 &= 2 \times {}^{10}C_4 p^4 q^6 \\
 {}^{10}C_5 p &= 2 \times {}^{10}C_4 q \\
 {}^{10}C_5 p &= 2 \times {}^{10}C_4 (1-p) \\
 252 p &= 420 (1-p) \\
 672 p &= 420
 \end{aligned}$$

$$\begin{aligned}
 \therefore p &= 0.625 \\
 \therefore q &= 1-p = 0.375
 \end{aligned}$$

$$\begin{aligned}
 N &= 10000 \\
 n &= 10; r = 0 \text{ (even)} \\
 p &= 0.625 \text{ (even)} \\
 q &= 0.375
 \end{aligned}$$

$$\begin{aligned}
 P(0 \text{ even}) &= {}^{10}C_0 (0.625)^0 (0.375)^{10} \\
 &= 5.499 \times 10^{-5}
 \end{aligned}$$

$$\begin{aligned}
 \text{No. of sets with 0 even} \\
 &= N \times P = 10000 \times 5.499 \times 10^{-5} \\
 &= 0.5499 \\
 &\approx 1
 \end{aligned}$$

$$\begin{aligned}
 (12) \quad (i) \quad n=3 \quad n=7 \\
 p = \frac{1}{11} \quad p = \frac{1}{21} \\
 q = \frac{10}{11} \quad q = \frac{20}{21}
 \end{aligned}$$

$$\begin{aligned}
 P(2 \text{ old \& no new}) &= P(2) \times P(0) \\
 &= {}^3C_2 \left(\frac{1}{11}\right)^2 \left(\frac{10}{11}\right)^1 \times {}^7C_0 \left(\frac{1}{21}\right)^0 \left(\frac{20}{21}\right)^7 \\
 &= 0.016
 \end{aligned}$$

$$\begin{aligned}
 (ii) \quad P(2 \text{ old and 0 new}) + P(2 \text{ new \& 0 old}) \\
 &= {}^3C_2 \left(\frac{1}{11}\right)^2 \left(\frac{10}{11}\right)^1 \times {}^7C_0 \left(\frac{1}{21}\right)^0 \left(\frac{20}{21}\right)^7 \\
 &\quad + {}^3C_0 \left(\frac{1}{11}\right)^0 \left(\frac{10}{11}\right)^3 \times {}^7C_2 \left(\frac{1}{21}\right)^2 \left(\frac{20}{21}\right)^5 \\
 &= 0.044
 \end{aligned}$$

$$\begin{aligned}
 (13) \quad (i) \quad n=7 \\
 r = 2, 3, 4, 5, 6, 7 \text{ (hit)} \\
 p = \frac{1}{4} \text{ (hit)} \\
 q = \frac{3}{4} \\
 P(\text{at least twice}) \\
 &= P(2) + P(3) + \dots + P(7) \\
 &= 1 - P(0) - P(1) \\
 &= 1 - {}^7C_0 \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^7 - {}^7C_1 \left(\frac{1}{4}\right)^1 \left(\frac{3}{4}\right)^6 \\
 &= 0.5551
 \end{aligned}$$

$$\begin{aligned}
 (ii) \quad n=? \\
 r = 1, 2, 3, 4, 5, \dots, n \\
 p = \frac{1}{4} \\
 q = \frac{3}{4} \\
 P(\text{hit at least once}) > \frac{2}{3} \\
 P(1) + \dots + P(n) > \frac{2}{3} \\
 1 - P(0) > \frac{2}{3} \\
 1 - {}^n C_0 \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^n > \frac{2}{3} \\
 \frac{1}{3} > \left(\frac{3}{4}\right)^n \\
 \therefore n = 4
 \end{aligned}$$

(5)

14)

$$n = ?$$

$$r = 2, 3, \dots \text{ (hits)}$$

$$p = \frac{1}{2} \text{ (hit)}$$

$$q = \frac{1}{2}$$

$$P(\text{destroy target}) > 0.99$$

$$P(2) + P(3) + \dots + P(n) > 0.99$$

$$1 - P(0) - P(1) > 0.99$$

$$1 - {}^n C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^n$$

$$- {}^n C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^{n-1} > 0.99$$

$$1 - \left(\frac{1}{2}\right)^n - n \left(\frac{1}{2}\right)^n > 0.99$$

$$0.01 > \left(\frac{1}{2}\right)^n (1+n)$$

$$\therefore n = \underline{\underline{11}}$$

15)

$$n = 5$$

$$P(1) = 0.4096$$

$$P(2) = 0.2048$$

$${}^5 C_1 p^1 q^4 = 0.4096$$

$${}^5 C_2 p^2 q^3 = 0.4096$$

$$\therefore p = \underline{\underline{0.2}}$$

$$\therefore q = 1 - p = \underline{\underline{0.8}}$$

16)

$${}^9 P(4) = P(2)$$

$${}^9 C_4 p^4 q^5 = {}^9 C_2 p^2 q^7$$

$${}^9 P^2 = q^2$$

$${}^9 P^2 = (1-p)^2$$

$$\therefore p = \underline{\underline{0.25}}$$

TYPE II

1) $np = 3$

$npq = 1.2$

$\therefore \frac{npq}{np} = \frac{1.2}{3}$

$\therefore q = \underline{0.4}$

$\therefore p = 1 - q = \underline{0.6}$

$\therefore n(0.6) = 3$
 $n = \underline{5}$

$P(X < 4) = P(0) + P(1) + P(2) + P(3)$

$= {}^5C_0 (0.6)^0 (0.4)^5 +$

${}^5C_1 (0.6)^1 (0.4)^4 +$

${}^5C_2 (0.6)^2 (0.4)^3 +$

${}^5C_3 (0.6)^3 (0.4)^2$

$= 0.663$

2) $np = 6$

$npq = 16$

$\therefore \frac{npq}{np} = \frac{16}{6}$

$\therefore q = \frac{16}{6} > 1$

Not possible

 \therefore False.

3) $np = 5$ & $n^2 p^2 + npq = 27$

$(5)^2 + npq = 27$

$npq = 2$

$\sigma = \sqrt{npq} = \sqrt{2}$
 $= 1.414$

x	f	fx	(6)
0	6	0	
1	20	20	
2	28	56	
3	12	36	
4	8	32	
5	6	30	
6	0	0	
	$N = 80$	174	

Mean $\bar{x} = \frac{\sum fx}{\sum f} = \frac{174}{80} = 2.175$

$\therefore npq = 2.175$

$6p^2 = 2.175$

$\therefore p = 0.3625$

$\therefore q = 1 - p = 0.6375$

$f = N \times {}^6C_x (0.3625)^x (0.6375)^{6-x}$

$0 \quad 5.37 \approx 5$

$1 \quad 18.32 \approx 18$

$2 \quad 26.04 \approx 26$

$3 \quad 19.75 \approx 20$

$4 \quad 8.42 \approx 8$

$5 \quad 1.92 \approx 2$

$6 \quad 0.18 \approx 0$

5)

x	$f = 96 \times {}^5C_x \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{5-x}$
0	3
1	15
2	30
3	30
4	15
5	3

6) Mean = $np = 4$ ⑦
 Var = $npq = \frac{4}{3}$
 $\therefore \frac{npq}{np} = \frac{1}{3} \quad \therefore q = \frac{1}{3} //$
 $\therefore p = 1 - q = \frac{2}{3} //$
 $\therefore n \left(\frac{2}{3}\right) = 4$
 $\therefore n = 6 //$
 $P(x \geq 1) = P(1) + \dots + P(6)$
 $= 1 - P(0)$
 $= 1 - {}^6C_0 \left(\frac{2}{3}\right)^0 \left(\frac{1}{3}\right)^6$
 $= 0.9986$

For observed mean & sd

x	f	fx	fx^2
0	3	0	0
1	15	15	15
2	30	60	60
3	30	90	270
4	15	60	240
5	3	15	15
	<u>96</u>	<u>275</u>	<u>909</u>

7) (i) Fair coin (unbiased)
 $N = 128 \quad n = 7$
 $p = \frac{1}{2} \quad q = \frac{1}{2}$

$f = N \times {}^7C_x \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{7-x}$

x	f
0	1
1	7
2	21
3	35
4	35
5	21
6	7
7	1

Mean \bar{x} (obs) = $\frac{\sum fx}{\sum f} = 2.865$

Mean \bar{x} (theoretical) = $np = 5 \left(\frac{1}{2}\right) = 2.5$

$\sigma_{(obs)} = \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2}$
 $= \sqrt{\frac{909}{96} - (2.865)^2} = 1.227$

$\sigma_{(the)} = \sqrt{npq} = \sqrt{5 \times \frac{1}{2} \times \frac{1}{2}} = 1.118$

\therefore Mean & SD of obs & theoretical are similar \therefore its Binomial

P.T.O.

(ii)

x	f_x	fx
0	7	0
1	6	6
2	19	38
3	35	105
4	30	120
5	23	115
6	7	42
7	1	7
	<u>$N = 128$</u>	<u>433</u>

$$\text{Mean } \bar{x} = \frac{\sum fx}{\sum f} = \frac{433}{128} = 3.383$$

$$\therefore 4p = 3.383$$

$$\therefore 7p = 3.383$$

$$\therefore p = \underline{0.483}$$

$$\therefore q = 1 - p = \underline{0.517}$$

x	$f = N \times {}^7C_x (0.483)^x (0.517)^{7-x}$
0	$1.26 \approx 1$
1	$8.26 \approx 8$
2	$23.16 \approx 23$
3	$36.06 \approx 36$
4	$33.69 \approx 34$
5	$18.89 \approx 19$
6	$5.88 \approx 6$
7	$0.78 \approx 1$

$$8) n = \text{even} = 2m \quad (8)$$

$$\text{Mode} = (n+1)P$$

$$= (2m+1)\left(\frac{1}{2}\right)$$

$$= m + \frac{1}{2} \quad \text{not integer}$$

$$\therefore \text{Mode} = m \ \&$$

$$= \frac{n}{2}$$

$$n = \text{odd} = (2m-1)$$

$$\text{Mode} = (n+1)P$$

$$= (2m-1+1)\frac{1}{2}$$

$$= m \quad \text{integer}$$

$$\therefore \text{Mode} = m \ \& \ m-1$$

$$= \frac{n+1}{2} \ \& \ \frac{n+1}{2} - 1$$

$$= \frac{n+1}{2} \ \& \ \frac{n-1}{2}$$